

# Recent Developments in Fracture Mechanics of Composite Materials Under Multi-Axial Stress

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## ABSTRACT

The study of fracture mechanics of composite materials under multi-axial stress conditions has become a critical area of research in materials science and engineering. Recent advancements have provided deeper insights into the complex interactions between different stress states, crack initiation, and propagation, as well as the overall material behavior under multi-axial loading. These developments are particularly relevant for industries such as aerospace, automotive, and civil engineering, where composite materials are extensively used for high-performance applications. This paper provides a comprehensive review of the latest progress in this field, focusing on key topics such as failure criteria, experimental testing methods, and numerical modeling approaches for predicting the fracture behavior of composites. Additionally, the paper addresses several ongoing challenges, including the accurate prediction of crack growth, the need for more effective failure models, and the enhancement of simulation efficiency for large-scale composite structures. Potential solutions and future directions for overcoming these challenges are also explored.

## KEYWORDS

Composite materials; Multi-axial stress; Fracture mechanics

## 1 Introduction

Composite materials are increasingly utilized in a wide range of engineering applications due to their excellent strength-to-weight ratio, durability, and superior performance under various loading conditions. These materials, particularly fiber-reinforced composites, are extensively used in industries such as aerospace, automotive, and civil engineering, where the need for high-performance materials is paramount. The inherent advantages of composites, including their high specific strength and stiffness, make them ideal candidates for structural applications where weight reduction and strength are critical.

However, despite their numerous benefits, the behavior of composite materials under multi-axial stress conditions remains complex and challenging to predict. This complexity arises mainly from the anisotropic nature of composites, where material properties depend on the fiber orientation, matrix properties, and loading direction. Such directional dependence can lead to unpredictable failure mechanisms, particularly under complex loading conditions involving multi-axial stress states, such as those encountered in real-world applications. Understanding the fracture behavior of composites under these conditions is essential for optimizing their design and ensuring their reliability in service.

Recent advancements in fracture mechanics have provided valuable insights into the failure mechanisms of composite materials. Notably, studies by Li and Sitnikova (2024)<sup>[1]</sup> and Zhang et al. (2024)<sup>[2]</sup> have focused on the application of the Hashin failure criterion, which addresses the interaction between fiber and matrix failure modes. Their work has contributed to the development of enhanced models for predicting damage progression in laminated composites under multi-axial stress states. These advancements are crucial for improving the design, performance, and safety of composite materials used in critical engineering applications, where multi-axial stress is often a key factor.

## 2 Fracture Mechanics of Composite Materials

Fracture mechanics plays a crucial role in understanding the failure behavior of composite materials. These materials are often subjected to complex loading conditions, where their failure depends not only on the material's intrinsic properties but also on the loading type and the interaction between different stress components. Several key parameters, including stress intensity factors (SIF), energy release rates (ERR), and the critical strain energy release rate ( $G_c$ ), are commonly used to characterize the fracture behavior of composite materials. These parameters provide insights into crack initiation, growth, and the overall failure process.

### 2.1 Stress Intensity Factor (SIF)

The stress intensity factor (SIF) is a fundamental parameter in fracture mechanics that quantifies the intensity of the stress field near the tip of a crack, which is critical in predicting crack propagation and material failure. It is especially

important in composite materials, where the SIF is influenced by a variety of factors such as the loading conditions, crack orientation, and the material's inherent anisotropy. Composite materials, being heterogeneous and anisotropic, exhibit directional properties that cause the SIF to vary significantly depending on the type of applied loading, whether tensile, shear, or a combination of both (mixed-mode loading). The complexity of composite materials, including the interactions between different layers or phases (e.g., fibers and matrix), further complicates the prediction of crack growth under multi-axial stress conditions.

For instance, under tensile loading, the stress at the crack tip may cause crack propagation along the fiber direction, while under shear or mixed-mode loading, the crack could propagate along interfaces or through the matrix. This variability makes predicting crack growth under multi-axial stress states particularly challenging, as different loading modes interact in complex ways, influencing crack initiation and propagation. Studies such as the work by Shao et al. (2021)<sup>[3]</sup> have demonstrated the importance of accurate SIF calculations in predicting failure behavior in laminated composites under these complex multi-axial loading conditions. Their research emphasized the need for precise modeling of the crack tip stress field to better predict failure mechanisms in composite materials and to improve the reliability of structural designs involving composites.

## 2.2 Energy Release Rate (ERR)

The energy release rate (ERR) is a crucial parameter in fracture mechanics, particularly for evaluating the stability of cracks in composite materials. It represents the amount of energy released from the system as a crack propagates, and it is used to predict the conditions under which a crack will grow or remain stable. In composite materials, the ERR is particularly important because it accounts for the energy available for crack propagation, which depends on the material's fracture toughness and the applied stress. As the crack length increases, the ERR quantifies the additional energy required to continue crack growth, providing critical insight into material failure mechanisms.

Under multi-axial loading conditions, the ERR becomes more complex due to the interactions between different stress components. The presence of normal stresses, shear stresses, and mixed-mode loading—where both tensile and shear stresses act simultaneously—can influence the ERR in ways that are not present under uniaxial loading. These interactions often result in mixed-mode crack propagation, where cracks may grow in multiple directions, complicating the prediction of failure behavior. To accurately model crack propagation in such scenarios, advanced analytical models are needed that can account for these interactions and provide reliable predictions of the material's response under complex loading.

The research by Vajari et al. (2022)<sup>[4]</sup> introduced a thermodynamically consistent approach to studying ductile fracture under multi-axial stress, which provides a valuable framework for understanding the ERR in ductile composite materials under complex loading conditions. Their work has contributed to enhancing our understanding of how ERR interacts with different stress components, improving the prediction of failure in composite materials subjected to multi-axial stress.

## 3 Failure Criteria for Composite Materials

Several failure criteria have been developed to predict the failure of composite materials under multi-axial stress conditions. These criteria play a vital role in understanding the behavior of composites under complex loading environments, enabling engineers to assess the material's limits and predict failure before it occurs. Each criterion is designed to capture different aspects of failure behavior, and the selection of the most appropriate one depends on the material's properties, the nature of the loading, and the specific application.

The Maximum Stress Criterion is one of the simplest and most commonly used failure criteria. It assumes that failure occurs when the stress in any direction exceeds the material's maximum allowable stress. This criterion is useful for quick assessments but may be overly simplistic, especially in cases where stress interaction effects are significant.

The Maximum Strain Criterion is based on the material's strain limits, predicting failure when the strain in any direction surpasses a critical value. This criterion is often used for ductile materials but can be less effective for composites, where failure often involves a combination of different failure modes such as matrix cracking and fiber breakage.

Hashin's Failure Criteria, widely applied to fiber-reinforced composites, combines the effects of normal and shear stresses to predict various failure modes, including fiber breakage, matrix cracking, and interfacial failure. It is particularly effective in capturing the failure mechanisms of unidirectional composite materials but may not fully account for the complexities of woven or random fiber composites.

Puck's Failure Criterion is more advanced and incorporates the interaction between different failure modes such as fiber and matrix failure under complex loading conditions. This criterion provides a more comprehensive approach, especially for woven fabrics, and is better suited for multi-axial loading scenarios. However, its complexity makes it computationally more demanding.

Each of these criteria has its strengths and limitations, and selecting the appropriate one requires a thorough

understanding of the composite material's behavior and the loading conditions it will experience in real-world applications.

## 4 Experimental Methods for Fracture Testing

Experimental methods are fundamental for evaluating the fracture behavior of composite materials under multi-axial stress conditions, as they provide the necessary data to understand how these materials perform when subjected to complex and realistic loading scenarios. The most commonly used testing techniques include tensile and compressive testing, shear testing, and multi-axial fatigue testing, each serving a specific purpose and providing unique insights into material performance.

Tensile and compressive tests are foundational in assessing a material's response to normal stresses. These tests help identify the material's ultimate tensile strength (UTS) and compressive strength, both of which are crucial parameters for applications involving significant axial loading. The tensile test, in particular, is important for determining how the material will perform under stretching forces, while the compressive test focuses on behavior under compressive forces, which is especially important for materials used in structural applications.

Shear testing, on the other hand, focuses on assessing the material's behavior when subjected to shear forces, which is particularly vital in understanding failure modes such as delamination or interfacial cracking in composite materials. These types of failures are common in layered composites, where the interface between the fiber and matrix plays a critical role in overall material strength.

Multi-axial fatigue testing is an advanced technique used to simulate real-world conditions in which materials experience cyclic loading in multiple directions simultaneously. This testing is crucial for understanding how composite materials perform under repetitive stress, revealing how damage accumulates over time and providing insights into fatigue life and durability.

Recent developments in experimental testing techniques have led to more accurate and reliable data collection, especially under complex loading conditions. Innovations in sensor technology, high-resolution imaging systems, and advanced data analysis methods now enable more precise measurement of crack initiation, propagation, and overall material degradation. These advancements are crucial for better understanding the complex interactions between normal and shear stresses and how these interactions influence failure mechanisms in composite materials. Studies like Vicentini et al. (2024)<sup>[5]</sup> have made significant contributions by enhancing fracture models, providing clearer insights into energy decomposition during brittle fracture under multi-axial stress.

## 5 Numerical Modeling Approaches

Numerical modeling is a powerful tool for simulating the fracture behavior of composite materials under multi-axial stress. With the increasing complexity of composite material structures, advanced numerical methods are crucial for understanding the failure mechanisms that occur under different loading conditions. Finite element analysis (FEA) is the most commonly used technique for modeling the stress distribution, crack initiation, and propagation in composite materials. FEA allows for the detailed simulation of material behavior, taking into account the anisotropic properties of composites and the interaction between various stress components. Recent advances in FEA techniques have greatly improved the accuracy and efficiency of simulations, enabling more precise predictions of failure in complex composite structures. These improvements have been particularly important in simulating the mechanical behavior of composites under multi-axial loading, as demonstrated in recent studies such as Chang et al. (2022)<sup>[6]</sup>, which explored the quasi-static mechanical behavior of filament-wound composite structures under tension, torsion, and multi-axial loading.

### 5.1 Homogeneous vs. Heterogeneous Modeling

In the numerical modeling of composite materials, two primary approaches are widely used: homogeneous and heterogeneous models. Each approach offers distinct advantages and limitations, depending on the complexity of the material structure and the application requirements.

Homogeneous models simplify the modeling process by treating the composite material as a single, isotropic material. This approach assumes that the composite behaves uniformly in all directions, disregarding the differences between its components, such as the fibers and matrix. The main advantage of homogeneous models lies in their simplicity and computational efficiency, making them suitable for preliminary design analyses or when the composite's microstructure does not significantly affect the material's overall behavior. However, this approach can be less accurate when applied to complex composite structures, as it fails to account for the interactions between different material phases, such as fiber-

matrix bonding, or the anisotropic nature of the composite. These limitations can lead to inaccurate predictions of failure modes, especially under multi-axial stress conditions where material heterogeneity plays a crucial role in determining the fracture behavior.

On the other hand, heterogeneous models explicitly account for the individual components of the composite, such as the fibers, matrix, and the interfaces between them. These models consider the interactions between the different phases, making them more accurate in simulating the behavior of composite materials under complex loading conditions. While more computationally demanding, heterogeneous models provide a more detailed and realistic representation of material behavior, especially under multi-axial stresses, where the interaction between fiber and matrix is critical for failure prediction. These models are essential for accurately capturing damage mechanisms such as delamination, matrix cracking, and fiber breakage.

## 5.2 Cohesive Zone Modeling

Cohesive zone modeling (CZM) has become an essential tool for simulating crack initiation and growth in composite materials, particularly in cases where failure occurs due to damage at the fiber-matrix interface. Unlike traditional fracture mechanics methods that focus on crack tip singularities, CZM explicitly models the degradation of material as cracks propagate through the interface between the fiber and matrix. This method introduces a traction-separation law, which describes the relationship between the stresses acting on the crack surface and the relative displacement of the crack faces. As the crack progresses, the material experiences progressive damage that can be modeled in terms of changes in the cohesive forces at the crack interface.

CZM is especially effective in capturing interfacial failure modes, which are crucial in composite materials, as the fiber-matrix interface significantly influences the overall strength and durability of the material. Delamination, matrix cracking, and fiber-matrix debonding are common failure mechanisms in composites, and CZM provides a framework to simulate these phenomena in a physically consistent manner. The ability to model these failures helps in understanding how cracks propagate in a multi-phase material system under various loading conditions.

One of the key advantages of CZM is its ability to simulate the material's progressive failure, including crack initiation, growth, and the eventual final fracture, without the need for mesh refinement around the crack tip. This approach makes it highly efficient for simulating large-scale structures, such as composite laminates and sandwich panels. Studies have shown that CZM accurately predicts the onset and progression of failure in composite materials, particularly in cases where interfacial damage plays a dominant role<sup>[7]</sup>.

## 6 Key Challenges and Future Directions

Despite significant progress in the study of fracture mechanics of composite materials under multi-axial stress, several key challenges continue to hinder the accurate prediction and effective application of these materials. One of the most pressing issues is the accurate prediction of crack initiation and propagation under complex loading conditions. Composite materials are inherently anisotropic, meaning their response to stress varies depending on the orientation of fibers and the type of loading. This, coupled with the interaction between normal and shear stresses under multi-axial loading, complicates the accurate prediction of failure. The heterogeneous nature of composite materials, with distinct fiber and matrix phases, further complicates the modeling of crack propagation, making it difficult to account for all the variables that influence failure.

Additionally, while several failure criteria, such as the Maximum Stress Criterion, Hashin's Failure Criterion, and Puck's Failure Criterion, are widely used, they often fall short in capturing the full range of failure behaviors observed in composites under multi-axial stress. Current models tend to focus on specific failure modes, such as fiber breakage or matrix cracking, but they may not fully account for the interactions between these modes, or for more complex failure mechanisms that arise under different stress states. This highlights the need for the development of more comprehensive and versatile failure criteria that can predict the combined effects of multiple failure modes under complex loading conditions.

Another significant challenge is improving the efficiency of numerical simulations, especially for large-scale composite structures. These simulations require considerable computational resources, particularly when modeling complex behaviors such as crack propagation in heterogeneous materials. As composite structures become increasingly large and complex, the demand for faster, more efficient simulation techniques grows.

Future research should focus on integrating experimental and numerical methods to develop more robust predictive models. By combining detailed experimental data with advanced computational models, researchers can improve the accuracy of failure predictions and develop more reliable design tools. This integrated approach is essential for optimizing the performance of composite materials and advancing their practical applications in critical engineering sectors such as

aerospace, automotive, and civil engineering.

## About the Author

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